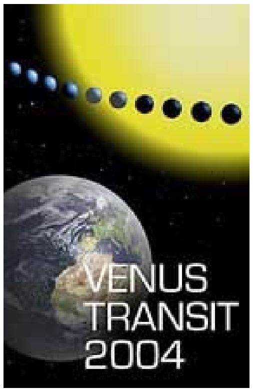


Analysis of the Observations of Contacts

The On-Line Calculation of the AU

Explanatory Note 1 - by Patrick Rocher and Jean-Eudes Arlot (IMCCE)

How can we calculate the value of the AU from a time measurement?

Before we describe the methods that were used, let's first come back to the nature of the measurement performed. This will help to understand the various considerations needed to ensure a proper functioning of the on-line calculation performed on June 8, 2004, at the time of the Venus transit event. This was a unique feature of the VT-2004 programme - nothing like this had ever been attempted before. The post-event calculations based on the entire data base are described in Explanatory Note 2.

Description of the project

We received a lot of e-mails showing that many observers did not fully understand the real problem and believe that measuring the Astronomical Unit (AU) by means of observation of the contacts is the same as measuring the length of an object by using a ruler. Then they take it that all the measurements are equivalent and that correspondingly the final result is just the average of all the measurements. In reality, it is not as simple as that.

The measurement of the AU by means of timing observations of contacts is not a measure of distance, but a measure of time, this time being different for all the observers. In fact, the phenomenon that we observe originates from the angle (the "parallax") between the direction towards the contact from the centre of the Earth (theoretically calculated) and the direction towards the contact from the observing site. The observed timing of the contact will then be used to calculate the corresponding value of the AU.

Moreover, in the VT-2004 Observing Campaign project, we did not select in advance the observed parallaxes since the observers would not move to specific, optimally located sites (i.e. locations where the parallax is at maximum) as did the astronomers of the past centuries. In the present case, the geographical distribution of the observers on the surface of the Earth is random and, in fact, concentrated in Europe….

So, for a calculation in real time of the AU, it is not possible to use the Delisle's method which associates two sites that are geographically well located. We should calculate the AU by comparing each timing observation to the theoretical value: which must be the corresponding value of the AU that makes the difference between the observed and calculated moments of contact as small as possible?

However, another critical problem now emerges for the real-time calculation, as this was planned on June 8 during the transit: the **convergence** of the algorithm used. Any observation that happens to be too far off the theoretical value (low accuracy, wrong contact assigned, etc.) may lead to an **infinitely large value of the AU** (i.e., putting the Sun at an infinitely large distance) and would therefore corrupt the average computed from many observations. Since we did not know in advance what would be the actual accuracy of the observations entered into the database in real time, it was impossible a priori to set up a criterion that would allow effectively to reject "bad" observations. Instead, we had to build **an algorithm with a strong constraint that ensures that it does not diverge, even in the case of "bad" observations**.

Hence, the algorithm for the on-line calculation of the AU works as follows :

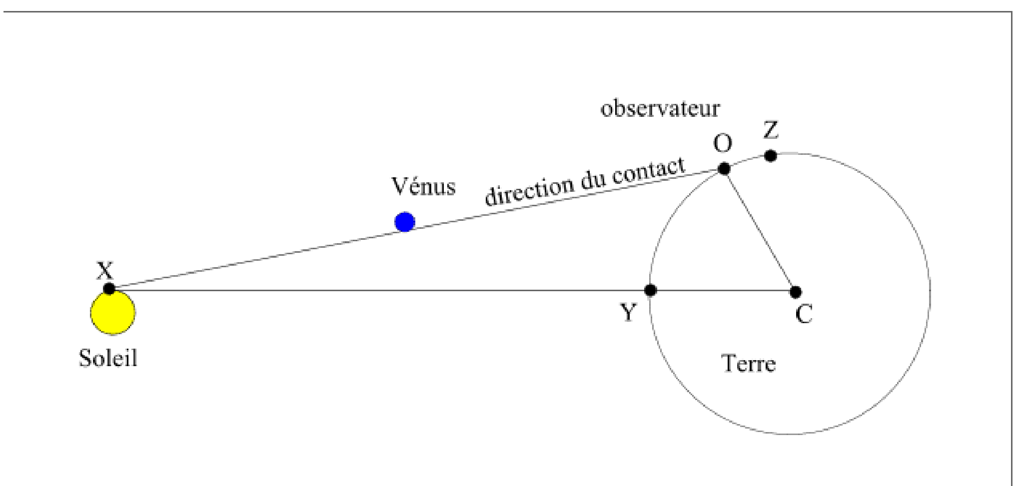
- on-line arrival of a first contact timing observation, calculation of the corresponding AU
- arrival of the second observation, calculation of the corresponding AU and then, averaging with the previously calculated AU
- and so on, the nth observation providing a calculated AU which will be averaged with the (n-1)th previously calculated AU.

Note in particular that it would not have been feasible to adapt/modify the rejection criteria in real-time as the timing data were received on June 8: we concluded that the only workable modus would be to accept all observations (that are within 30 minutes of the correct time) and then to make sure that the « bad » observations did not derail the on-line calculation.

What do we measure and what do we calculate ?

The problem that we solve is the calculation of the diurnal parallax of a point of contact as seen from a given observing site by measuring the shift in time between the observed and the theoretical timing of the contact.

The figure below explains the geometry of the problem :



The diurnal parallax is the angle OXC

On this figure, **O** is the observer, **OX** is the line of sight towards the contact and the line **XC** joins the point of contact **X** on the limb of the Sun with the centre **C** of the Earth. Astronomers refer to the angle **OXC** as the "diurnal parallax".

The predictions are made using the value of the solar parallax π_0=8,794142", i.e., using a value of the AU perfectly known nowadays, AU = 149597870 km. The prediction t_c of the time of the contact consists in finding, knowing the size of the triangle **XOC** thanks to the knowledge of the AU, the time when the contact occurs. Mathematically, we have a relationship R (a function) which, for any observing site, provides a value of t_c from the value of the AU. This function depends on numerous physical parameters such as the diameters of the Sun and Venus expressed in km, the geographical position of the observer, the equatorial radius of the Earth and the flatness of our planet, as well as the spatial position of the centres of Venus and the Sun at any time.

The calculation of the AU from the observation of the timing of a contact then consists in the inverse operation (in mathematics, the reciprocal function R^{-1}): what would be the value of the AU in km so that the calculated time of the contact t_c (using the relationship R) is exactly the same as the observed value ?

In this way it is then possible, theoretically, to calculate a corresponding value of the AU in real time for each contact timing observation received.

How are the individually calculated values of the AU then used ?

The only possible calculation to be made in real time when receiving the observations on-line is the averaging of the calculated AU of individual observations and a statistical parameter that expresses the uncertainty, i.e., the standard deviation.

It is therefore necessary to be aware of the difference between the measurements made by the observers – the contact timings – and the corresponding calculation of the AU. In other words, we did not measure the AU directly (as with a ruler), but the value of the AU (e.g., in km) is the result of a calculation based upon the measurement of time.

An important note in order to understand the next steps : What is a ”good” measurement ?

Intuitively, a “good” measurement is a measurement that is as close to the reality as possible and which did not “place” the Sun at an infinite distance! The experience of the past centuries leads us to believe that an error of about one minute of time of a contact timing may be common among lay observers. However, as we are now in the XXIst century (use of GPS, recording of CCD images, use of good optics, but less experience of the lay observers than of their professional colleagues during earlier transit events, …) the circumstances have certainly changed. Nevertheless, before the observations on June 8, we did not know what would be the actual mean error of the measurements. This could therefore not be of any help for the real time calculation.

Moreover, suppose for a second that we receive a set of “good” measurements : may we then expect that these measurements also will lead to a good value of the AU ?

Unfortunately the answer is “not always”: it depends on the contact involved and on the site of observation. We can explain this in two ways :

- First from geometrical considerations. On the figure above, it is evident that the form of the triangle **XOC** depends on the position of the observer **O**. If the observer is in **Y**, the angle **OXC** (the diurnal parallax) is zero: the triangle is completely flat and it is impossible to determine a corresponding value of the AU (the diurnal parallax is zero and the calculation does not depend on the value of the AU). Contrarily, if the observer is at **Z**, the angle **OXC** is at its maximum (the angle **XOC** is 90°), i.e., the diurnal parallax is as large as possible. The form of the triangle characterizes the geometry of the problem. Note that we did not measure the diurnal parallax but the effect of the diurnal parallax on the calculation of the contact times. The form of the triangle characterizes the geometry of the problem but the effect of the diurnal parallax may be zero even if the diurnal parallax is at maximum. Thus, for a given geometrical contact, all the sites on Earth defined by the intersection of the shadow cone (or penumbra depending on the contact) and the terrestrial ellipsoid will see the contact at the same time as a fictive observer located at the center of the Earth. So, the closer an observer is situated to these sites, the smaller will be the effect of the diurnal parallax.

A good observing site must thus fulfil two criteria : to have a diurnal parallax as large as possible and to be as far as possible from the intersection of the shadow cone (or penumbra) and the terrestrial ellipsoid. The worst site of observation on Earth for this method is the point of this line of intersection which has a diurnal parallax equal to zero, i.e. from where the point of contact is directly above in the sky, at zenith. For each contact, there is only one site for which this is the case. In conclusion, a "good" observing site must be far from this line of intersection and must therefore see the contact when the Sun is as low as possible above the local horizon.

- Secondly, from the linear approximate formula that gives the variation of the parallax from the variation of the time shift.

$$(A.\cos\varphi\cos\lambda + B.\cos\varphi\sin\lambda + C.\sin\varphi)\delta\pi_0 = -\frac{dD}{dt}(t_0 - t_c)$$

For each contact, the coefficients *A*, *B*, *C* and the value *dD/dt* are fixed.

For example, for the first internal contact, we have:

A = 2,1970, *B* = 0,2237, *C* = 1,1206, *dD/dt* = –2,9394"/min.

Let $X = (A.\cos\varphi\cos\lambda + B.\cos\varphi\sin\lambda + C.\sin\varphi)$

The variation of the parallax for a given shift (t_o-t_c) is the quotient of 2,9394 by X. However, this coefficient is a function of the coordinates of the observer: it approaches zero when the observer approaches the points of the Earth whose geographical coordinates are solution of the equation X=0. Thus for the same variation of observation in time, one will obtain very important differences for the variation of the parallax and thus of the AU. The value of the AU even becomes infinite when X is null, even with a very good observation!

We can also indicate the geometrical difference between a direct and a linearized calculation. The X=0 equation has, as a solution, the intersection of a plane passing through the centre of the Earth and the terrestrial ellipsoid; the real solution is the intersection of a cone with the terrestrial ellipsoid, the linearization thus replaces the cone by a tangent plan to a cone passing through the centre of the Earth.

The solution of the « constrained » system

We saw above that it is possible, theoretically, to calculate a value of the AU for any observed timing t_o of a contact made from any site of observation. However, we also saw that it is, in fact, impossible to do so for timings made at some sites, since these timings – however good they may be – will lead to an infinite and hence, unacceptable, value for the AU. For timings which are removed from the correct value by several minutes of time (as might be commonplace according to the experience of the past centuries), the algorithm for the calculation of the reciprocal function does not converge.

We would like to take into account all the received observations in order to produce a sliding average as the observations arrive. In order to be sure that this calculation does not begin to diverge because of some "bad" timings, we solve a slightly different system in which we have introduced a constraint in order to prevent the calculated value of the AU from becoming infinite. Of course, we did not constrain totally the system (i.e. we did not impose the value of the AU) but we forced the triangle **XOC** to remain a finite triangle (i.e. to prevent the Sun and Venus to move ad infinitum). For that, in the calculation of the reciprocal function providing the value of the AU depending on the observed timing, we multiplied the vectors Earth – Venus and Earth – Sun by the ratio CAU/EAU, where CAU is the calculated AU and EAU the estimated AU. Such a system converges towards the value of the AU as does the system without such a constraint, but it has the great advantage of having a finite solution for all the possible values permitted during the acquisition of the data (up to 30 minutes error on the timings). However, convergence of the calculation also requires that the observer did not observe at the zenith and this was fortunately not the case for any of the timings received.

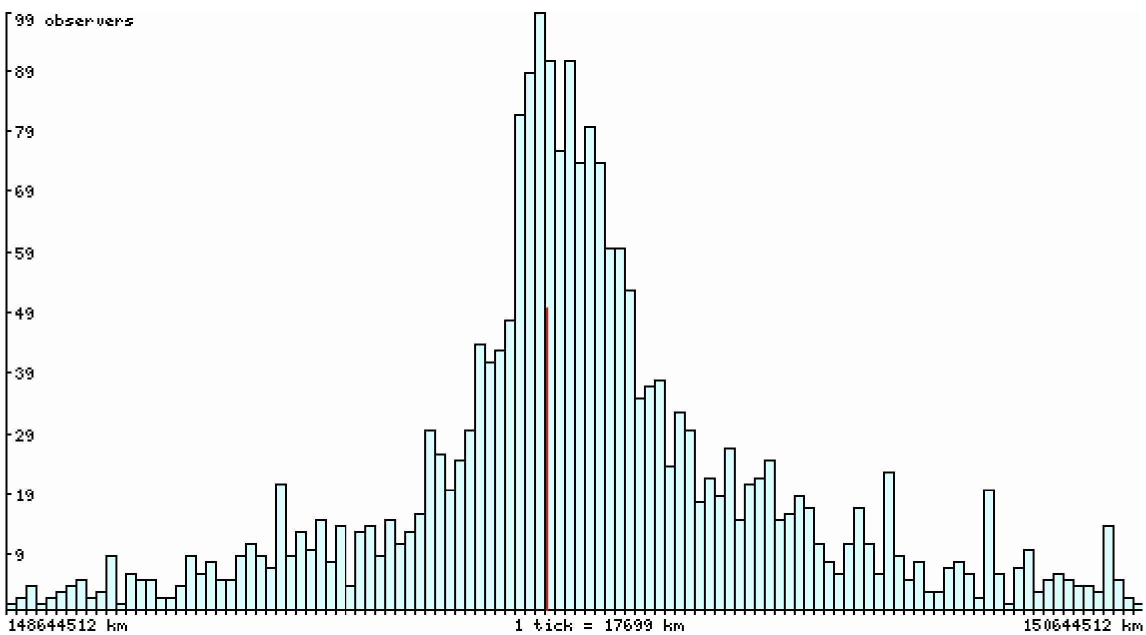
Normally, if the observations are realistic, i.e., if the observers send timings they have really measured, the average value of the AU as deduced with the constrained system is the same as that would have been found with a corresponding non-constrained system in which the very bad observations have been rejected. However, this does not hold if some observers would enter "fantasy" timings, just to test the algorithm (trying to make the calculation diverge !). We noted that some entries of this kind were made after June 18, while the database was still accepting new data and the average of the calculated AU was accordingly degraded.

So we may say that the overall effect of the constrained system was in fact to improve artificially the "quality" of the observations for the calculation of the AU. However, as in the non-constrained method, the geographical site of observation still played an important role and it is not possible to compare results from one site to another, neither in the constrained system nor in the non-constrained.

Evolution of the calculated average and dispersion of the AU before June 18

In conclusion, we provided to the observers with real-time individual results, the errors of which appeared to be better than what would have been the case if we had used a non-constrained system (but still comparable in relative terms, from one observer to another) since the average of the AU was realistic. We could have provided the difference between the observed timing and the true one to the observers, but preferred not to do so, since it was still possible to enter a new observation and some observers might then have been tempted to cheat…

We show above the plot of the evolution of the average of the AU (for the four contacts) as we received the data (green) and the evolution of the dispersion as the root mean square of the residuals σ (blue). They include data received before June 18, a date on which we noticed that some "false" data were being entered by some observers, apparently to "test" the system.



Distribution of the calculated value of the AU using all timings in the database

The distribution of the calculated values of the AU from the entire database used for the averaging shows that it follows a normal law and is very nearly Gaussian. If the obviously false values are rejected, the distribution is even better. It is thus justifiable to assume a distribution according to a normal law and hence, the average of the calculated AU is a good estimate of the resulting AU. It is also possible to calculate the true dispersion of this mean value by dividing the Gaussian dispersion by the square root of the number of observations.

We show below the final results obtained with the constrained system, by removing obviously wrong entries from the database and using all observations with a timing error of less than 30 minutes of time, in all 4367 observations from 1440 observers:

Contact	Number of observations	Mean value of AU (km)	Diff. from true AU (km)	Standard deviation (km)	Parallax
T_1	722	149908018	+310148	76755	8,775936"
T_2	1139	149076728	-521142	1445676	8,824890"
T_3	1336	149438938	-158932	38558	8,803500"
T_4	1170	149840793	+242923	135072	8,779890"
All	4367	149529684	-68186	55059	8,798158"
Average $(T_1+T_2+T_3+T_4)/4$		149566119	-31751		8,796015"

The final result of these calculations, based on 4367 timings from all four contacts, is thus

1 AU = 149 529 684 km +/- 55 059km

The difference of this value from the "true" value of the AU is **-68 186 km**.

We finally note that in the course of the real-time calculation, we published the average of the calculated AU for each contact and not the general average of all the observations. That led to a curious effect at the beginning of the event: when an observer erroneously entered an observed value of the first or second contact as if it belonged to a contact that had not yet taken place yet (the third or the fourth), it produced gave an individual, false result for this contact, which entered into the calculated average value of the AU. Afterwards, this bad result was progressively "averaged out" as more data concerning this contact arrived and its disturbing influence was rapidly reduced.

We will see in Explanatory Note 2, that the non-constrained system provides results very similar to the ones obtained with the above described constrained system, when using only timings which fall within intervals of 16 and 8 seconds of time, respectively, around the predicted (theoretical) timings of the contacts.